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SOLUTION OF HELMHOLTZ' EQUATION FOR A HALF-PLANE WITH BOUNDARY CONDITIONS CONTAINING HIGH ORDER DERIVATIVES

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The problem of the steady-state acoustic oscillation is examined in a fluid the surface of which is covered by an infinitely thin elastic body (a membrane, a plate, a shell). Properties of the cover are given by means of a differential operator of arbitrary order with constant coefficients. A solution of the problem is formulated for arbitrary sources (point or distributed) which are located both in the fluid and on the cover.

Notation

P — pressure, f — extraneous body force in the fluid, F — extraneous surface force, ρ — density of fluid, ρ_0 — density of covering material, μ — surface density of coverage, E — Young's modulus, σ — Poisson's modulus, T — membrane tension, $2h$ — thickness of coverage, ω — circular frequency, k — wave number in the fluid.

The time factor $e^{-i\omega t}$ is omitted everywhere.

1. Formulation of the problem. Examples. Problem related to the influence of thin elastic objects (membranes, plates, shells) on acoustic processes in a fluid are at present of urgent interest. Mathematical boundary value problems which arise in the investigation of such effects as a rule have a specific feature: differential operators which are involved in the definition of boundary conditions have a higher order than the order of the equation itself.

Let the lower half-plane $y > 0$ be filled with a compressible fluid. Processes in this fluid will be described in terms of pressure P. For $y > 0$ we shall assume that the inhomogeneous Helmholtz equation is satisfied

$$\Delta P + k^2 P = \operatorname{div} f \qquad (1.1)$$

We shall seek a solution of this equation for the following boundary conditions:

$$L\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) P \equiv m_1\left(-i\frac{\partial}{\partial x}\right)\frac{\partial P}{\partial y} + m_2\left(-i\frac{\partial}{\partial x}\right)P =$$

$$= m_1\left(-i\frac{\partial}{\partial x}\right) f_y + \sum_{s=1}^{l} q_s\left(-i\frac{\partial}{\partial x}\right) F_s \quad (y=0) \qquad (1.2)$$

Extraneous effects (sources) $\operatorname{div} f$, $f_y\,(y=0)$ and $F_1, \ldots F_l$ are assumed to be specified functions; we shall assume that they are localized in some finite volume. In other words, $\operatorname{div} f\,(x, y)$, $f_y(x, 0)$ and $F_1(x), \ldots, F_l(x)$ are classical or generalized functions of their own arguments. These functions are equal to zero outside a certain finite domain (so-called functions with a finite carrier). We can consider that the components of vector $f(x, y)$ are also specified. It is however not necessary to impose limitations on the domain in which they are different from zero. The quantity k is assumed to be a complex number with positive real and imaginary parts (i.e. the fluid medium turns out to be absorbing). The case of positive real values k should be considered as a result of a passage to the limit of $\operatorname{Im} k^2 \to +0$ (principle of limiting absorption). Operators $m_s(-i\partial/\partial x)$ and $q_s(-i\partial/\partial x)$ are polynomials of argument $-i\partial/\partial x$; coefficients of these polynomials do not depend on x. In the following some limitations will be placed on algebraic properties of polynomials.

The solution will be sought in the class of functions which together with their derivatives of arbitrary order show exponential decay with respect to x and y with increasing distance from the domain occupied by sources.

The fact that derivatives with respect to y of higher order than first do not enter into boundary condition (1.2), does not imply a limitation of generality. Derivatives with respect to y of second and higher order can be eliminated from (1.2) by means of the Helmholtz equation (1.1).

We shall present the most prevalent examples of boundary condition (1.2).

1. The surface of fluid is free (the case of absence of cover)

$$P = F_y \qquad (1.3)$$

2. The surface of the fluid is rigidly fixed

$$\partial P/\partial y = f_y \qquad (1.4)$$

3. "Impedance" boundary condition

$$\partial P/\partial y + \alpha P = f_y + \alpha F_y \qquad (1.5)$$

As was noted in [1], this condition arises especially when the fluid is separated from a completely rigid body by a thin elastic layer of thickness $2h$. In this case it turns out that

$$\alpha = 2h\rho\omega^2 \frac{(1+\sigma)(1-2\sigma)}{E(1-\sigma)}$$

4. A membrane is located on the surface of the fluid

$$\frac{1}{\rho\omega^2}\left[T\left(-i\frac{\partial}{\partial x}\right)^2 - \mu\omega^2\right]\frac{\partial P}{\partial y} + P = \frac{1}{\rho\omega^2}\left[T\left(-i\frac{\partial}{\partial x}\right)^2 - \mu\omega^2\right] f_y + F_y \qquad (1.6)$$

5. An elastic plate separates two identical fluids (occupying the lower $y > 0$ and upper $y < 0$ half-plane, respectively). The plate is assumed to be capable of bending

as well as symmetrical motions. Separating the symmetrical and antisymmetrical part of functions which describe the processes in the system, we can break up the general problem into two independent ones each of which is formed for the lower half-plane. We have

$$P^{\pm}(x, y) = {}^1/_2\,[P(x, y) \pm P(x, -y)]$$

$$f_x^{\pm}(x, y) = {}^1/_2\,[f_x(x, y) \pm f_x(x, -y)], \qquad f_y^{\pm}(x, y) = {}^1/_2\,[f_y(x, y) \mp f_y(x, -y)]$$

$$F_x^{\pm}(x) = {}^1/_2\,[F_{1,x}(x) \pm F_{2,x}(x)], \qquad F_y^{\pm}(x) = {}^1/_2\,[F_{1,y}(x) \mp F_{2,y}(x)]$$

Here $F_1(F_2)$ represents the force acting on the lower (upper) surface of the plate. Body forces are not taken into consideration.

By examining the symmetrical part we can arrive at the boundary condition

$$\frac{1}{\rho\omega^2}\left[3\left(\frac{E}{1-\sigma^2}\right)\left(-i\frac{\partial}{\partial x}\right)^2 - \rho_0\omega^2\right]\frac{\partial P^+}{\partial y} + h\left[\left(-i\frac{\partial}{\partial x}\right)^2 - \rho_0\omega^2\frac{(1+\sigma)(1-2\sigma)}{E(1-\sigma)}\right]P^+ =$$
$$= \frac{1}{\rho\omega^2}\left[\frac{E}{1-\sigma^2}\left(-i\frac{\partial}{\partial x}\right)^2 - \rho_0\omega^2\right]f_y^+ + h\left[\left(-i\frac{\partial}{\partial x}\right)^2 - \rho_0\omega^2\frac{(1+\sigma)(1-2\sigma)}{E(1-\sigma)}\right]F_y^+ -$$
$$-\frac{\sigma}{1-\sigma}\frac{\partial F_x^+}{\partial x} \tag{1.7}$$

For the antisymmetrical part we have, correspondingly,

$$\frac{1}{\rho\omega^2}\left[\frac{Eh^3}{1-\sigma^2}\left(-i\frac{\partial}{\partial x}\right)^4 - \rho_0 h\omega^2\right]\frac{\partial P^-}{\partial y} + P^- =$$
$$= \frac{1}{\rho\omega^2}\left[\frac{Eh^3}{3(1-\sigma^2)}\left(-i\frac{\partial}{\partial x}\right)^4 - \rho_0 h\omega^2\right]f_y^- + F_y^- + h\frac{\partial F_x^-}{\partial x} \tag{1.8}$$

6. The plate, as before, is capable of bending as well as symmetrical motions and is located on the surface of the fluid. The boundary condition here has a quite cumbersome form

$$\frac{1}{\rho\omega^2}\left[\frac{E}{1-\sigma^2}\left(-i\frac{\partial}{\partial x}\right)^2 - \rho_0 h\omega^2\right]\left[\frac{Eh^3}{3(1-\sigma^2)}\left(-i\frac{\partial}{\partial x}\right)^4 - \rho_0 h\omega^2\right]\frac{\partial P}{\partial y} +$$
$$+\left\{\left[\frac{E}{1-\sigma^2}\left(-i\frac{\partial}{\partial x}\right)^2 - \rho_0 h\omega^2\right] + h\left[\frac{Eh^3}{3(1-\sigma^2)}\left(-i\frac{\partial}{\partial x}\right)^4 - \rho_0 h\omega^2\right]\left[\left(-i\frac{\partial}{\partial x}\right)^2 +\right.\right.$$
$$\left.\left.+\frac{\rho_0\omega^2(1+\sigma)(1-2\sigma)}{E(1-\sigma)}\right]\right\}\frac{P}{2} = \frac{1}{\rho\omega^2}\left[\frac{E}{1-\sigma^2}\left(-i\frac{\partial}{\partial x}\right)^2 - \rho_0 h\omega^2\right]\left[\frac{Eh^3}{3(1-\sigma^2)}\left(-i\frac{\partial}{\partial x}\right)^4 -\right.$$
$$\left.-\rho_0 h\omega^2\right]f_y + \left[\frac{E}{1-\sigma^2}\left(-i\frac{\partial}{\partial x}\right)^2 - \rho_0 h\omega^2\right]\left(F_y^- + h\frac{\partial F_x^-}{\partial x}\right) + \left[\frac{Eh^3}{3(1-\sigma^2)}\left(-i\frac{\partial}{\partial x}\right)^4 -\right.$$
$$\left.-\rho_0 h\omega^2\right]\left\{h\left[\left(-i\frac{\partial}{\partial x}\right)^2 - \frac{\rho_0\omega^2(1+\sigma)(1-2\sigma)}{E(1-\sigma)}\right]F_y^+ - \frac{\sigma}{1-\sigma}\frac{\partial F_x^+}{\partial x}\right\} \tag{1.9}$$

If by virtue of some considerations (see [2]) we can neglect the symmetrical (antisymmetrical) motion of the plate, the problem simplifies and the boundary relationship takes a form analogous to (1.8) or (1.7), respectively, and differs from it only in doubling of some coefficients.

At present time problems for moving covers are also examined [3]. In this case derivatives of uneven order would have participated in operators $m_1(-i\partial/\partial x)$ and $m_2(-i\partial/\partial x)$.

2. Field of sources located in the fluid. The general problem (1.1), (1.2) can be broken up into two independent problems assuming inhomogeneous Helmholtz' equation (1.1) and homogeneous boundary condition in the first problem

$$\Delta P + k^2 P = \operatorname{div} f \quad (y > 0), \qquad LP = 0 \quad (y = 0) \tag{2.1}$$

and the converse in the second problem. At first let us consider the first problem and let us construct its solution with the aid of Green's function. Green's function $G(x, y, x_0, y_0)$ represents a field of a point source of the δ-function type located in the depth of the fluid (field of a pulsating cylinder). We have

$$\Delta G + k^2 G = \delta(x - x_0, y - y_0) \quad (y > 0), \qquad LG = 0 \qquad (y = 0) \tag{2.2}$$

We shall look for function G in the form of a sum of two terms, the first of which (G_0) represents the field of a point source for a boundless fluid medium and the second (G_1) the reflected field which is due to boundary condition (2.2) for $y = 0$

$$G = G_0 + G_1 \tag{2.3}$$

Both terms are written in the form of an expansion in plane waves

$$G_0 = \frac{1}{4\pi i} \int_{-\infty}^{\infty} \frac{1}{\sqrt{k^2 - \lambda^2}} e^{i\lambda(x-x_0)+i\sqrt{k^2-\lambda^2}\,|y-y_0|} d\lambda$$

$$G_1 = \frac{1}{2\pi i} \int_{-\infty}^{\infty} g(\lambda) e^{i\lambda x + i\sqrt{k^2-\lambda^2}\,y} d\lambda \tag{2.4}$$

The branch of the radical is fixed by the requirement $\operatorname{Im}\sqrt{k^2 - \lambda^2} > 0$ for $\operatorname{Im}\lambda = 0$. The function $g(\lambda)$ is sought on the basis of the boundary condition (2.2) for $y = 0$. We have

$$\frac{1}{2\pi i} \int_{-\infty}^{\infty} \left[g(\lambda) l(\lambda) + \frac{1}{2\sqrt{k^2 - \lambda^2}} e^{-i\lambda x_0 + iy_0\sqrt{k^2-\lambda^2}} l^\circ(\lambda) \right] e^{i\lambda x} d\lambda = 0 \tag{2.5}$$

$$l(\lambda) = i\sqrt{k^2 - \lambda^2}\, m_1(\lambda) + m_2(\lambda), \qquad l^\circ(\lambda) = -i\sqrt{k^2 - \lambda^2}\, m_1(\lambda) + m_2(\lambda) \tag{2.6}$$

Now restriction 1 is placed on boundary operator L. The algebraic function $l(\lambda)$ does not have real roots. Solving (2.5) with respect to $g(\lambda)$ we obtain

$$g(\lambda) = -\frac{1}{2\sqrt{k^2 - \lambda^2}} \frac{l^\circ(\lambda)}{l(\lambda)} e^{-i\lambda x_0 + iy_0\sqrt{k^2-\lambda^2}} \tag{2.7}$$

From this it follows in turn

$$G = \frac{1}{4\pi i} \int_{-\infty}^{\infty} \frac{1}{\sqrt{k^2 - \lambda^2}\, l(\lambda)} e^{i\lambda(x-x_0)} \left[l(\lambda) e^{i|y-y_0|\sqrt{k^2-\lambda^2}} - l^\circ(\lambda) e^{i(y+y_0)\sqrt{k^2-\lambda^2}} \right] d\lambda \tag{2.8}$$

We recall that quantity k^2 is considered complex. In case of $\operatorname{Im} k = 0$ in problems described in Section 1, real roots $l(\lambda)$ may occur (they correspond to so-called surface waves). The integral for G becomes in this case divergent and its regularization is achieved by means of introduction into k of the positive imaginary part with subsequent realization of passage to the limit of $\operatorname{Im} k \to +0$. As a result of such a procedure, expressions arise in which integration along the real axis is replaced by an integration over a contour close to the real axis. This contour lies in the complex plane and bypasses in a special manner singular points of the expression under the integral (see for example [4]).

It is not difficult to become convinced that the constructed function G satisfies conditions at infinity. In fact the expression DG (where D is an arbitrary operator of differentiation with respect to x and y) splits into two terms of the following form:

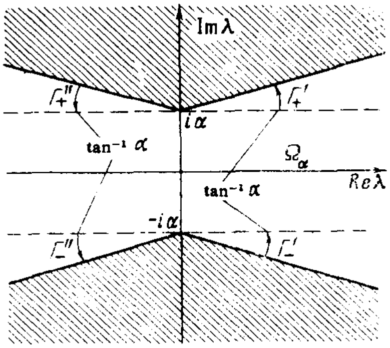

Fig. 1

$$\int_{-\infty}^{\infty} \frac{r(\lambda)}{\sqrt{k^2-\lambda^2}\, l(\lambda)} e^{i\lambda X + i\sqrt{k^2-\lambda^2}\,Y} d\lambda \qquad (2.9)$$

where $Y \geq 0$, and $r(\lambda)$ is some polynomial of arguments λ and $\sqrt{k^2-\lambda^2}$. As singular points of the expression under the integral, points of bifurcation $\lambda = \pm k$ and poles in roots $l(\lambda)$ appear. The number of these singular points is finite, in addition to this and in accordance with the assumption they are not located on the real axis. Therefore we can find such an $\alpha > 0$ in the region Ω_α defined by the inequality

$$|\operatorname{Im}\lambda| \leqslant \alpha + \alpha\,|\operatorname{Re}\lambda|$$

(it is left without hatching in Fig. 1), that there will be no singular points. Further, by virtue of the fact that $\operatorname{Im}\sqrt{k^2-\lambda^2} > 0$ on the real axis while $\lim \operatorname{Im}\sqrt{k^2-\lambda^2} = \pm\infty$ for $\lambda \to \pm\infty$, this α can be selected in such a manner that everywhere in Ω_α we will have $\operatorname{Im}\sqrt{k^2-\lambda^2} > \alpha$. Finally, the expression $r(\lambda)/\sqrt{k^2-\lambda^2}\, l(\lambda)$ has an algebraic character and does not have singularities in the closed region Ω_α. Therefore the following uniform estimate must be applicable in Ω_α

$$\left|\frac{r(\lambda)}{\sqrt{k^2-\lambda^2}\, l(\lambda)}\right| < A(|\mu|^\gamma + 1) \qquad (A,\ \gamma = \mathrm{const} > 0,\ \mu = \operatorname{Re}\lambda)$$

For the sake of definiteness we shall now assume that $X > 0$. The contour of integration in (2.9) is displaced upward in such a manner that it will coincide with rays Γ'_+ and Γ''_+. The integral with respect to Γ'_+ is estimated as follows:

$$\left|\int_{\Gamma'_+} \frac{r(\lambda)}{\sqrt{k^2-\lambda^2}\, l(\lambda)} e^{i\lambda X + i\sqrt{k^2-\lambda^2}\,Y} d\lambda\right| < A\int_0^\infty (\mu^\lambda + 1)\, e^{-\mu\alpha X - \alpha X - \alpha Y}\sqrt{1+\alpha^2}\, d\mu =$$
$$= A\sqrt{1+\alpha^2}\, e^{-\alpha X - \alpha Y}\left[\Gamma(\gamma+1)(\alpha X)^{-\gamma-1} + \frac{1}{\alpha X}\right] \qquad (2.10)$$

The same expression also estimates the integral with respect to Γ''_+.

In a similar manner the cases $X > 0$ and $X = 0$ can be examined. By the same token it can be considered as proved that the constructed function $G(x, y, x_0, y_0)$ satisfies the conditions at infinity and consequently is the Green's function of our problem.

We note that restriction 1 has in some respects a necessary character. Namely: no regularization of diverging integral (2.11) in the case of real roots $l(\lambda)$ (for $\operatorname{Im} k > 0$) satisfies stated conditions at infinity. (Various regularizations would differ in the quantity expressed by deduction of the function under the integral in the root $l(\lambda)$, but this does not give the decay for $Y = \mathrm{const}$ at $X \to \pm\infty$).

The constructed Green's function possesses certain symmetry properties with respect to the point of observation (x, y) and the point (x_0, y_0) in which the source is located. In physics such properties are referred to as a reciprocity principle. Comparing derivatives of G with respect to x and x_0 in (2.8), we have the following for the arbitrary operator of differentiation $D(-i\partial/\partial x)$:

$$D\left(-i\frac{\partial}{\partial x}\right) G(x, y, x_0, y_0) = D\left(i\frac{\partial}{\partial x_0}\right) G(x, y, x_0, y_0) \qquad (2.11)$$

It is evident that with respect to variables (x_0, y_0) Green's function must satisfy Equations

$$\left(\frac{\partial^2}{\partial x_0^2} + \frac{\partial^2}{\partial y_0^2} + k^2\right) G(x, y, x_0, y_0) = \delta(x - x_0, y - y_0) \qquad (y_0 > 0) \tag{2.12}$$

$$L\left(-\frac{\partial}{\partial x_0}, \frac{\partial}{\partial y_0}\right) G(x, y, x_0, y_0) = 0 \qquad (y_0 = 0) \tag{2.13}$$

If the following condition is satisfied

$$L\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) = L\left(-\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) \tag{2.14}$$

as is the case in examples of Section 1, then Green's function is simply symmetrical

$$G(x, y, x_0, y_0) = G(x_0, y_0, x, y)$$

and therefore the point of observation and the point where the source is located can be interchanged. In case when derivatives of odd order with respect to x are present in L (case of moving covers), it is necessary in transposition of the points of observation and the source to change the velocity of the cover to the reverse.

We note that function G_1 represents some analog of the field of imaginary representation of the source which is located in the point $(x_0, -y_0)$.

Knowledge of Green's function permits to find the field of an arbitrary system of sources located in the fluid. Solution of problem (2.1) is expressed through G by means of convolution operation

$$P(x, y) = \iint_B G(x, y, x_0, y_0) \operatorname{div} f(x_0, y_0)\, dx_0\, dy_0 \tag{2.15}$$

This operation has meaning because according to assumption the region B (this is the designation of the carrier of function $\operatorname{div} f$) is finite (see [5]).

3. Field of sources located on the surface. Now we shall turn to the second part of our general problem

$$\Delta P + k^2 P = 0 \quad (y > 0), \qquad LP = m_1\left(-i\frac{\partial}{\partial x}\right) f_y + \sum_{s=1}^{l} q_s\left(-i\frac{\partial}{\partial x}\right) F_s \qquad (y = 0) \tag{3.1}$$

Its solution is most easily achieved by means of Green's "boundary" function $H(x, y, x_0)$ which satisfies conditions

$$\Delta H + k^2 H = 0 \qquad (y > 0), \qquad LH = \delta(x - x_0) \qquad (y = 0) \tag{3.2}$$

It is not difficult to obtain the following expression for H:

$$H(x, y, x_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{l(\lambda)} e^{i\lambda(x - x_0) + i\sqrt{k^2 - \lambda^2}\, y}\, d\lambda \tag{3.3}$$

Solution of problem (3.1) is expressed through the following convolution

$$P(x, y) = \int_a^b H(x, y, x_0)\left[m_1\left(-i\frac{\partial}{\partial x_0}\right) f_{y_0}(x_0, 0) + \sum_{s=1}^{l} q_s\left(-i\frac{\partial}{\partial x_0}\right) F_s(x_0)\right] dx_0 \tag{3.4}$$

Here $[a, b]$ is some interval outside of which functions $f_y(x, 0), F_1(x), F_2(x), \ldots F_l(x)$ are identically equal to zero.

Apparently there must exist a connection between functions G and H. We shall prove below that $H(x, y, x_0)$ with one additional restriction on operator L can be expressed through $G(x, y, x_0, y_0)$ in a local manner (by means of differentiation operation and passage to the limit for $y_0 \to +0$) and write the solution of the general equation in terms of one function G.

We let y_0 in (2. 8) go to zero

$$G(x, y, x_0, 0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{m_1(\lambda)}{l(\lambda)} e^{i\lambda(x-x_0)+i\sqrt{k^2-\lambda^2}y} d\lambda \tag{3.5}$$

Applying operator L we obtain (setting $y = 0$)

$$L\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)\Big|_{y=0} G(x, y, x_0, 0) = m_1\left(-i\frac{\partial}{\partial x}\right)\delta(x - x_0) \tag{3.6}$$

It is now evident that the solution of the problem

$$\Delta P + k^2 P = 0 \quad (y > 0), \qquad LP = m_1\left(-i\frac{\partial}{\partial x}\right) f_y(x, 0) \quad (y = 0) \tag{3.7}$$

can be written by means of G with the aid of convolution in the following manner:

$$P(x, y) = \int_a^b G(x, y, x_0, 0) f_y(x_0, 0)\, dx_0 \tag{3.8}$$

This result becomes quite obvious if in a comparison with (2, 15) it is taken into account that the jump of the normal component of any vector is its surface divergence.

Now we shall apply the Green's function G a certain linear differential operator N

$$N\left(-\frac{\partial}{\partial x_0}, \frac{\partial}{\partial y_0}\right) = n_1\left(i\frac{\partial}{\partial x_0}\right)\frac{\partial}{\partial y_0} + n_2\left(i\frac{\partial}{\partial x_0}\right) \tag{3.9}$$

Here n_s are some polynomials of their own argument and coefficients of these polynomials (as before with m_s and q_s) are assumed to be independent of coordinates.

Carrying out the differentiation and setting $y_0 = 0$, we obtain

$$N\left(-\frac{\partial}{\partial x_0}, \frac{\partial}{\partial y_0}\right)\Big|_{y_0=0} G(x, y, x_0, y_0) =$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{m_1(\lambda)\, n_2(\lambda) - m_2(\lambda)\, n_1(\lambda)}{l(\lambda)} e^{i\lambda(x-x_0)+i\sqrt{k^2-\lambda^2}y} d\lambda \tag{3.10}$$

Now we apply operator L to NG and put $y = 0$

$$L\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)\Big|_{y=0} N\left(-\frac{\partial}{\partial x_0}, \frac{\partial}{\partial y_0}\right)\Big|_{y_0=0} G(x, y, x_0, y_0) =$$

$$= \left[m_1\left(-i\frac{\partial}{\partial x}\right) n_2\left(-i\frac{\partial}{\partial x}\right) - m_2\left(-i\frac{\partial}{\partial x}\right) n_1\left(-i\frac{\partial}{\partial x}\right)\right]\delta(x - x_0) \tag{3.11}$$

We apply now restriction 3. 1 on operator L.

Polynomials $m_1(\lambda)$ and $m_2(\lambda)$ do not have common roots. In other words, the largest common denominator of these polynomials is a constant.

Then, according to a well-known theorem of algebra (see for example [6]) for any $m_1(\lambda)$ and $m_2(\lambda)$, we can select (by an innumerable multitude of methods) such polynomials $n_1(\lambda)$ and $n_2(\lambda)$ that the following relationship is satisfied

$$m_1(\lambda)\, n_2(\lambda) - m_2(\lambda)\, n_1(\lambda) = 1 \tag{3.12}$$

In advance we shall imply N to be such an operater that n_1 and n_2 satisfy the relationship (3, 12). Then

$$L\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)\Big|_{y=0} N\left(-\frac{\partial}{\partial x_0}, \frac{\partial}{\partial y_0}\right)\Big|_{y_0=0} G(x, y, x_0, y_0) = \delta(x - x_0) \tag{3.13}$$

From this it follows that

$$H(x, y, x_0) = N\left(-\frac{\partial}{\partial x_0}, \frac{\partial}{\partial y_0}\right)_{y_0=0} G(x, y, x_0, y_0) \tag{3.14}$$

and this proves the assertion about the possibility of local representation of H through G; this indicates that an arbitrary surface force applied to any point of the cover can be replaced in its action by some point source of multiple character located in the fluid in the immediate vicinity of the cover. This result is of definite interest because the action of various surface forces may not express themselves one through the other in a local mannei. Thus in example 5, Section 1 (Expression (1.7)) the tangential point force F_x^+ of the δ-function type can be replaced in its action only by some distributed normal force F_y^+, and conversely, the normal point force F_y^+ equal to $\delta(x-x_0)$ can be replaced only by a distributed tangential force. This is related to the circumstance that the solution of Equation

$$h\left[\left(-i\frac{\partial}{\partial x}\right)^2-\rho_0\omega^2\frac{(1+\sigma)(1-2\sigma)}{E(1-\sigma)}\right]F_y^+=\frac{\sigma}{1-\sigma}\frac{\partial F_x^+}{\partial x} \qquad (3.15)$$

with respect to any unknown F_x^+ and F_y^+ can be realized only by means of nonlocal operation, i. e. integration.

The general solution of the problem formulated in Section 1 can now be written in terms of one function G.

$$P(x,y)=\iint_B G(x,y,x_0,y_0)\operatorname{div} f(x_0,y_0)\,dx_0\,dy_0+\int_a^b G(x,y,x_0,0)\,f_y(x_0,0)\,dx_0+$$

$$+\int_a^b N\left(-\frac{\partial}{\partial x_0},\frac{\partial}{\partial y_0}\right)\Big|_{y_0=0} G(x,y,x_0,y_0)\sum_{s=1}^{l} q_s\left(-i\frac{\partial}{\partial x_0}\right)F_s(x_0)\,dx_0 \qquad (3.16)$$

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